

Transmitter Optimization in MISO Broadcast Channel with Common and Secret Messages

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Abstract—In this paper, we consider transmitter optimization in multiple-input single-output (MISO) broadcast channel with common and secret messages. The secret message is intended for K users and it is transmitted with perfect secrecy with respect to J eavesdroppers which are also assumed to be legitimate users in the network. The common message is transmitted at a fixed rate R_0 and it is intended for all K users and J eavesdroppers. The source operates under a total power constraint. It also injects artificial noise to improve the secrecy rate. We obtain the optimum covariance matrices associated with the common message, secret message, and artificial noise, which maximize the achievable secrecy rate and simultaneously meet the fixed rate R_0 for the common message.

keywords: Physical layer security, MISO, common and secret messages, secrecy rate, artificial noise, multiple eavesdroppers.

I. INTRODUCTION

The concept of achieving perfect secrecy using physical layer techniques was first introduced in [1] on a degraded wiretap channel. Later, this work was extended to more general broadcast channel and Gaussian channel in [2] and [3], respectively. Achieving secrecy using physical layer techniques as opposed to cryptographic techniques does not rely on the computational limitation of the eavesdroppers. Wireless networks can be easily eavesdropped due to the broadcast nature of the information transmission. With the growing applications on wireless networks, there is a growing demand for achieving secrecy on these networks. Secrecy in multi-antenna point-to-point wireless links has been studied by several authors, e.g., [4]–[8]. In [2], simultaneous transmission of a private message to receiver 1 at rate R_1 and a common message to receivers 1 and 2 at rate R_0 for two discrete memoryless channels with common input was considered. Recently, the work in [2] has been extended to multiple-input multiple-output (MIMO) broadcast channel with confidential and common messages in [9]–[11]. Motivated by the above works, in this paper, we consider transmitter optimization in multiple-input single-output (MISO) broadcast channel with common and secret messages. The secret message is intended for K users and it is transmitted with perfect secrecy with respect to J eavesdroppers which are also assumed to be legitimate users in the network. The common message is transmitted at a fixed rate R_0 and it is intended for all K users and J eavesdroppers. The source operates under a total power constraint. It also injects artificial noise to improve the secrecy rate. Under these settings, we obtain the optimum covariance matrices associated with the common

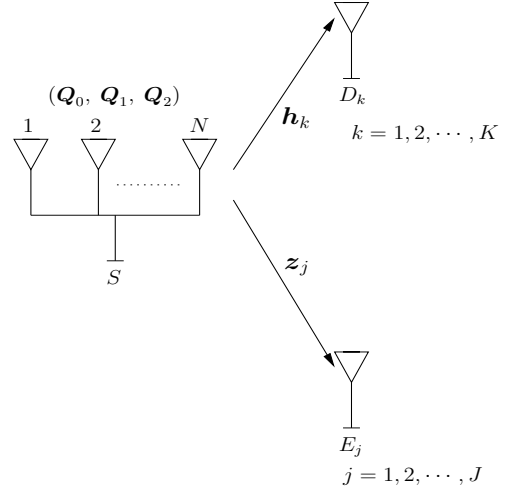


Fig. 1. System model for MISO broadcast channel with common and secret messages.

message, secret message, and artificial noise, which maximize the achievable secrecy rate and simultaneously meet the fixed rate R_0 for the common message. We note that the secrecy rate maximization in MISO channel without common message and in the presence of multiple eavesdroppers has been considered in [12] where the secret message is intended only for a single user (i.e., $K = 1$).

Notations: $\mathbf{A} \in \mathbb{C}^{N_1 \times N_2}$ implies that \mathbf{A} is a complex matrix of dimension $N_1 \times N_2$. $\mathbf{A} \succeq \mathbf{0}$ implies that \mathbf{A} is a positive semidefinite matrix. Complex conjugate transpose operation is denoted by $[\cdot]^*$. $\mathbb{E}[\cdot]$ denotes the expectation operator, and $\|\cdot\|$ denotes the 2-norm operator.

II. SYSTEM MODEL

Consider a MISO broadcast channel as shown in Fig. 1 which consists of a source S having N transmit antennas, K users $\{D_1, D_2, \dots, D_K\}$ each having single antenna, and J eavesdroppers $\{E_1, E_2, \dots, E_J\}$ each having single antenna. The complex channel gain from S to D_k is denoted by $\mathbf{h}_k \in \mathbb{C}^{1 \times N}$, $1 \leq k \leq K$. Likewise, the complex channel gain from S to E_j is denoted by $\mathbf{z}_j \in \mathbb{C}^{1 \times N}$, $1 \leq j \leq J$. We assume that eavesdroppers are non-colluding [8].

Let P_T denote the total transmit power budget in the system, i.e., the source S operates under total power constraint P_T . The communication between the source and the users and eavesdroppers happens in n channel uses. The source S transmits

two independent messages W_0 and W_1 , which are equiprobable over $\{1, 2, \dots, 2^{nR_0}\}$ and $\{1, 2, \dots, 2^{nR_1}\}$, respectively. W_0 is the common message to be conveyed to all D_k s and E_j s at information rate R_0 . W_1 is the secret message which has to be conveyed to all D_k s at some rate R_1 with perfect secrecy with respect to all E_j s [8]. For each W_0 drawn equiprobably from the set $\{1, 2, \dots, 2^{nR_0}\}$, the source maps W_0 to an i.i.d. ($\sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_0)$) codeword $\{\mathbf{X}_i^0\}_{i=1}^n$ of length n , where each $\mathbf{X}_i^0 \in \mathbb{C}^{N \times 1}$ and $\mathbf{Q}_0 = \mathbb{E}[\mathbf{X}_i^0 \mathbf{X}_i^{0*}]$. Similarly, for each W_1 drawn equiprobably from the set $\{1, 2, \dots, 2^{nR_1}\}$, the source, using a stochastic encoder, maps W_1 to an i.i.d. ($\sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_1)$) codeword $\{\mathbf{X}_i^1\}_{i=1}^n$ of length n , where each $\mathbf{X}_i^1 \in \mathbb{C}^{N \times 1}$ and $\mathbf{Q}_1 = \mathbb{E}[\mathbf{X}_i^1 \mathbf{X}_i^{1*}]$. The source also injects i.i.d. ($\sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_2)$) artificial noise sequence $\{\mathbf{X}_i^2\}_{i=1}^n$ of length n , where each $\mathbf{X}_i^2 \in \mathbb{C}^{N \times 1}$ and $\mathbf{Q}_2 = \mathbb{E}[\mathbf{X}_i^2 \mathbf{X}_i^{2*}]$. In the i th channel use, $1 \leq i \leq n$, the source transmits the sum of the symbols which is $\mathbf{X}_i^0 + \mathbf{X}_i^1 + \mathbf{X}_i^2$. Since the source is power limited, this implies that

$$\text{trace}(\mathbf{Q}_0) + \text{trace}(\mathbf{Q}_1) + \text{trace}(\mathbf{Q}_2) \leq P_T. \quad (1)$$

In the following, we will use \mathbf{X}^0 , \mathbf{X}^1 and \mathbf{X}^2 to denote the symbols in the codewords $\{\mathbf{X}_i^0\}_{i=1}^n$ and $\{\mathbf{X}_i^1\}_{i=1}^n$, and the artificial noise sequence $\{\mathbf{X}_i^2\}_{i=1}^n$, respectively. We also assume that all the channel gains are known and remain static over the codeword transmit duration. Let y_{D_k} and y_{E_j} denote the received signals at D_k and E_j , respectively. We have

$$y_{D_k} = \mathbf{h}_k \mathbf{X}^0 + \mathbf{h}_k \mathbf{X}^1 + \mathbf{h}_k \mathbf{X}^2 + \eta_{D_k}, \quad \forall k = 1, 2, \dots, K, \quad (2)$$

$$y_{E_j} = \mathbf{z}_j \mathbf{X}^0 + \mathbf{z}_j \mathbf{X}^1 + \mathbf{z}_j \mathbf{X}^2 + \eta_{E_j}, \quad \forall j = 1, 2, \dots, J, \quad (3)$$

where the η s are the noise components, assumed to be i.i.d. $\mathcal{CN}(0, N_0)$.

III. TRANSMITTER OPTIMIZATION IN MISO BROADCAST CHANNEL

Since the symbol \mathbf{X}^0 is transmitted at information rate R_0 irrespective of \mathbf{X}^1 , treating \mathbf{X}^1 as noise in (2), D_k s will be able to decode \mathbf{X}^0 if $\forall k = 1, 2, \dots, K$,

$$I(\mathbf{X}^0; y_{D_k}) = \log_2 \left(1 + \frac{\mathbf{h}_k \mathbf{Q}_0 \mathbf{h}_k^*}{N_0 + \mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^* + \mathbf{h}_k \mathbf{Q}_2 \mathbf{h}_k^*} \right) \geq R_0. \quad (4)$$

Similarly, treating \mathbf{X}^1 as noise in (3), E_j s will be able to decode \mathbf{X}^0 if $\forall j = 1, 2, \dots, J$,

$$I(\mathbf{X}^0; y_{E_j}) = \log_2 \left(1 + \frac{\mathbf{z}_j \mathbf{Q}_0 \mathbf{z}_j^*}{N_0 + \mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^* + \mathbf{z}_j \mathbf{Q}_2 \mathbf{z}_j^*} \right) \geq R_0. \quad (5)$$

Using (2) and with the knowledge of the symbol \mathbf{X}^0 , the information rate for \mathbf{X}^1 at D_k is

$$I(\mathbf{X}^1; y_{D_k} | \mathbf{X}^0) = \log_2 \left(1 + \frac{\mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*}{N_0 + \mathbf{h}_k \mathbf{Q}_2 \mathbf{h}_k^*} \right). \quad (6)$$

Similarly, using (3) and with the knowledge of \mathbf{X}^0 , the information rate for \mathbf{X}^1 at E_j is

$$I(\mathbf{X}^1; y_{E_j} | \mathbf{X}^0) = \log_2 \left(1 + \frac{\mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*}{N_0 + \mathbf{z}_j \mathbf{Q}_2 \mathbf{z}_j^*} \right). \quad (7)$$

A. Transmitter optimization - without artificial noise

In this subsection, we consider transmitter optimization in MISO broadcast channel when no artificial noise is injected by the source. Subject to the constraints in (1), (4) and (5), the achievable secrecy rate for \mathbf{X}^1 is obtained by solving the following optimization problem [8]:

$$R_1 = \max_{\mathbf{Q}_0, \mathbf{Q}_1} \min_{\substack{k=1,2,\dots,K \\ j=1,2,\dots,J}} \left\{ I(\mathbf{X}^1; y_{D_k} | \mathbf{X}^0) - I(\mathbf{X}^1; y_{E_j} | \mathbf{X}^0) \right\} \quad (8)$$

$$= \max_{\mathbf{Q}_0, \mathbf{Q}_1} \min_{\substack{k=1,2,\dots,K \\ j=1,2,\dots,J}} \log_2 \left(\frac{1 + \frac{\mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*}{N_0}}{1 + \frac{\mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*}{N_0}} \right) \quad (9)$$

$$= \log_2 \max_{\mathbf{Q}_0, \mathbf{Q}_1} \min_{\substack{k=1,2,\dots,K \\ j=1,2,\dots,J}} \left(\frac{N_0 + \mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*}{N_0 + \mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*} \right) \quad (10)$$

s.t.

$$\log_2 \left(1 + \frac{\mathbf{h}_k \mathbf{Q}_0 \mathbf{h}_k^*}{N_0 + \mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*} \right) \geq R_0, \quad \forall k = 1, 2, \dots, K, \quad (11)$$

$$\log_2 \left(1 + \frac{\mathbf{z}_j \mathbf{Q}_0 \mathbf{z}_j^*}{N_0 + \mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*} \right) \geq R_0, \quad \forall j = 1, 2, \dots, J, \quad (12)$$

$$\mathbf{Q}_0 \succeq \mathbf{0}, \quad \mathbf{Q}_1 \succeq \mathbf{0}, \quad \text{trace}(\mathbf{Q}_0) + \text{trace}(\mathbf{Q}_1) \leq P_T. \quad (13)$$

The constraints (11) and (12) are obtained from (4) and (5), respectively. The objective function in (8) is obtained from (6) and (7). We rewrite the optimization problem in (10) in the following equivalent form:

$$\max_{\mathbf{Q}_0, \mathbf{Q}_1} \min_{\substack{k=1,2,\dots,K \\ j=1,2,\dots,J}} \left(\frac{N_0 + \mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*}{N_0 + \mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*} \right) \quad (14)$$

s.t.

$$\left(1 + \frac{\mathbf{h}_k \mathbf{Q}_0 \mathbf{h}_k^*}{N_0 + \mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*} \right) \geq 2^{R_0}, \quad \forall k = 1, 2, \dots, K,$$

$$\left(1 + \frac{\mathbf{z}_j \mathbf{Q}_0 \mathbf{z}_j^*}{N_0 + \mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*} \right) \geq 2^{R_0}, \quad \forall j = 1, 2, \dots, J,$$

$$\mathbf{Q}_0 \succeq \mathbf{0}, \quad \mathbf{Q}_1 \succeq \mathbf{0}, \quad \text{trace}(\mathbf{Q}_0) + \text{trace}(\mathbf{Q}_1) \leq P_T. \quad (15)$$

Further, we rewrite the innermost minimization in (14), namely,

$$\min_{\substack{k=1,2,\dots,K \\ j=1,2,\dots,J}} \left(\frac{N_0 + \mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*}{N_0 + \mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*} \right), \quad (16)$$

in the following equivalent maximization form:

$$\max_t t \quad (17)$$

$$\text{s.t.} \quad t(N_0 + \mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*) - (N_0 + \mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*) \leq 0, \quad \forall k = 1, 2, \dots, K, \quad \forall j = 1, 2, \dots, J. \quad (18)$$

Substituting the above maximization form in (14), we get the following single maximization form:

$$\begin{aligned} & \max_{\mathbf{Q}_0, \mathbf{Q}_1, t} t \quad (19) \\ \text{s.t. } & \forall k = 1, 2, \dots, K, \quad \forall j = 1, 2, \dots, J, \\ & t(N_0 + \mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*) - (N_0 + \mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*) \leq 0, \\ & (2^{R_0} - 1)(N_0 + \mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*) - (\mathbf{h}_k \mathbf{Q}_0 \mathbf{h}_k^*) \leq 0, \\ & (2^{R_0} - 1)(N_0 + \mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*) - (\mathbf{z}_j \mathbf{Q}_0 \mathbf{z}_j^*) \leq 0, \\ & \mathbf{Q}_0 \succeq \mathbf{0}, \quad \mathbf{Q}_1 \succeq \mathbf{0}, \quad \text{trace}(\mathbf{Q}_0) + \text{trace}(\mathbf{Q}_1) \leq P_T. \quad (20) \end{aligned}$$

For a given t , the above problem is formulated as the following semidefinite feasibility problem [13]:

$$\text{find } \mathbf{Q}_0, \mathbf{Q}_1, \quad (21)$$

subject to the constraints in (20). The maximum value of t , denoted by t_{max} , can be obtained using bisection method as follows. Let t_{max} lie in the interval $[t_{ll}, t_{ul}]$. The value of t_{ll} can be taken as 1 (corresponding to the minimum secrecy rate of 0) and t_{ul} can be taken as $(1 + \min_{k=1,2,\dots,K} \frac{P_T \|\mathbf{h}_k\|^2}{N_0})$, which corresponds to the minimum information capacity among D_k s when the entire power P_T is allotted to the source S . Check the feasibility of (20) at $t = (t_{ll} + t_{ul})/2$. If feasible, then $t_{ll} = t$, else $t_{ul} = t$. Repeat this until $t_{ul} - t_{ll} \leq \zeta$, where ζ is a small positive number. Using t_{max} in (10), the secrecy rate is given by

$$R_1 = \log_2 t_{max}. \quad (22)$$

Remark: We note that the maximum common message information rate, R_0^{max} , can be obtained as follows:

$$\begin{aligned} R_0^{max} &= \max_{\mathbf{Q}_0} \min_{\substack{k=1,2,\dots,K \\ j=1,2,\dots,J}} \{I(\mathbf{X}^0; y_{D_k}), I(\mathbf{X}^0; y_{E_j})\} \quad (23) \\ \text{s.t. } & \mathbf{Q}_0 \succeq \mathbf{0}, \quad \text{trace}(\mathbf{Q}_0) \leq P_T, \quad (24) \end{aligned}$$

where $I(\mathbf{X}^0; y_{D_k})$ and $I(\mathbf{X}^0; y_{E_j})$ in (23) are obtained from (4) and (5), respectively, with $\mathbf{Q}_1 = \mathbf{Q}_2 = \mathbf{0}$. The above optimization problem can be easily solved using the method as proposed above to solve (10). Also, using the K.K.T conditions, it can be shown that R_0^{max} attains its maximum value when $\text{trace}(\mathbf{Q}_0) = P_T$, i.e., when all the available power is used. This implies that for $R_1 > 0$, $R_0 < R_0^{max}$.

B. Rank-1 approximation of \mathbf{Q}_1 and \mathbf{Q}_0 - without artificial noise

The optimal solutions \mathbf{Q}_0 and \mathbf{Q}_1 obtained from (19) may or may not have rank 1. This can be easily seen from the K.K.T conditions of the optimization problem (19). For practical application, a rank-1 approximation of \mathbf{Q}_0 and \mathbf{Q}_1 can be done as follows. Let $\phi^0 \in \mathbb{C}^{N \times 1}$ and $\phi^1 \in \mathbb{C}^{N \times 1}$ be the unit norm eigen directions of \mathbf{Q}_0 and \mathbf{Q}_1 corresponding to the largest eigen values, respectively. We take $P_0 \phi^0 \phi^{0*}$ and $P_1 \phi^1 \phi^{1*}$ as the rank-1 approximation of \mathbf{Q}_0 and \mathbf{Q}_1 , respectively, where $P_0 \geq 0$, $P_1 \geq 0$ and $P_0 + P_1 \leq P_T$. We substitute $\mathbf{Q}_0 = P_0 \phi^0 \phi^{0*}$ and $\mathbf{Q}_1 = P_1 \phi^1 \phi^{1*}$ in the

optimization problem (19), which results in the following optimization problem:

$$\begin{aligned} & \max_{P_0, P_1, t} t \quad (25) \\ \text{s.t. } & \forall k = 1, 2, \dots, K, \quad \forall j = 1, 2, \dots, J, \\ & t(N_0 + P_1 \mathbf{z}_j \phi^1 \phi^{1*} \mathbf{z}_j^*) - (N_0 + P_1 \mathbf{h}_k \phi^1 \phi^{1*} \mathbf{h}_k^*) \leq 0, \\ & (2^{R_0} - 1)(N_0 + P_1 \mathbf{h}_k \phi^1 \phi^{1*} \mathbf{h}_k^*) - (P_0 \mathbf{h}_k \phi^0 \phi^{0*} \mathbf{h}_k^*) \leq 0, \\ & (2^{R_0} - 1)(N_0 + P_1 \mathbf{z}_j \phi^1 \phi^{1*} \mathbf{z}_j^*) - (P_0 \mathbf{z}_j \phi^0 \phi^{0*} \mathbf{z}_j^*) \leq 0, \\ & P_0 \geq 0, \quad P_1 \geq 0, \quad P_0 + P_1 \leq P_T. \quad (26) \end{aligned}$$

For a given t , the above problem is formulated as the following linear feasibility problem:

$$\text{find } P_0, P_1, \quad (27)$$

subject to the constraints in (26). The maximum value of t can be obtained using the bisection method and the corresponding secrecy rate can be obtained using (22).

C. Transmitter optimization - with artificial noise

In this subsection, we consider transmitter optimization in MISO broadcast channel when artificial noise is injected by the source. Subject to the constraints in (1), (4) and (5), the achievable secrecy rate for \mathbf{X}^1 is obtained by solving the following optimization problem:

$$\begin{aligned} R_1 &= \max_{\mathbf{Q}_0, \mathbf{Q}_1, \mathbf{Q}_2} \min_{\substack{k=1,2,\dots,K \\ j=1,2,\dots,J}} \{I(\mathbf{X}^1; y_{D_k} | \mathbf{X}^0) - I(\mathbf{X}^1; y_{E_j} | \mathbf{X}^0)\} \quad (28) \\ &= \max_{\mathbf{Q}_0, \mathbf{Q}_1, \mathbf{Q}_2} \min_{\substack{k=1,2,\dots,K \\ j=1,2,\dots,J}} \log_2 \left(\frac{1 + \frac{\mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*}{N_0 + \mathbf{h}_k \mathbf{Q}_2 \mathbf{h}_k^*}}{1 + \frac{\mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*}{N_0 + \mathbf{z}_j \mathbf{Q}_2 \mathbf{z}_j^*}} \right) \quad (29) \\ &= \log_2 \max_{\mathbf{Q}_0, \mathbf{Q}_1, \mathbf{Q}_2} \min_{\substack{k=1,2,\dots,K \\ j=1,2,\dots,J}} \left(\frac{N_0 + \mathbf{h}_k \mathbf{Q}_2 \mathbf{h}_k^* + \mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*}{N_0 + \mathbf{h}_k \mathbf{Q}_2 \mathbf{h}_k^*} \right) \left(\frac{N_0 + \mathbf{z}_j \mathbf{Q}_2 \mathbf{z}_j^*}{N_0 + \mathbf{z}_j \mathbf{Q}_2 \mathbf{z}_j^* + \mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*} \right) \quad (30) \end{aligned}$$

s.t.

$$\forall k = 1, 2, \dots, K, \quad \forall j = 1, 2, \dots, J, \quad \log_2 \left(1 + \frac{\mathbf{h}_k \mathbf{Q}_0 \mathbf{h}_k^*}{N_0 + \mathbf{h}_k \mathbf{Q}_2 \mathbf{h}_k^* + \mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*} \right) \geq R_0, \quad (31)$$

$$\log_2 \left(1 + \frac{\mathbf{z}_j \mathbf{Q}_0 \mathbf{z}_j^*}{N_0 + \mathbf{z}_j \mathbf{Q}_2 \mathbf{z}_j^* + \mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*} \right) \geq R_0, \quad (32)$$

$$\begin{aligned} & \mathbf{Q}_0 \succeq \mathbf{0}, \quad \mathbf{Q}_1 \succeq \mathbf{0}, \quad \mathbf{Q}_2 \succeq \mathbf{0}, \\ & \text{trace}(\mathbf{Q}_0) + \text{trace}(\mathbf{Q}_1) + \text{trace}(\mathbf{Q}_2) \leq P_T, \quad (33) \end{aligned}$$

where the constraints (31) and (32) are obtained from (4) and (5), respectively, and the objective function in (28) is obtained from (6) and (7). We rewrite the optimization problem in (30)

in the following equivalent form:

$$\begin{aligned}
& \max_{\mathbf{Q}_0, \mathbf{Q}_1, \mathbf{Q}_2} \min_{\substack{k=1,2,\dots,K \\ j=1,2,\dots,J}} \left(\frac{N_0 + \mathbf{h}_k \mathbf{Q}_2 \mathbf{h}_k^* + \mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*}{N_0 + \mathbf{h}_k \mathbf{Q}_2 \mathbf{h}_k^*} \right) \\
& \left(\frac{N_0 + \mathbf{z}_j \mathbf{Q}_2 \mathbf{z}_j^*}{N_0 + \mathbf{z}_j \mathbf{Q}_2 \mathbf{z}_j^* + \mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*} \right) \quad (34) \\
& \text{s.t. } \forall k = 1, 2, \dots, K, \quad \forall j = 1, 2, \dots, J, \\
& \left(1 + \frac{\mathbf{h}_k \mathbf{Q}_0 \mathbf{h}_k^*}{N_0 + \mathbf{h}_k \mathbf{Q}_2 \mathbf{h}_k^* + \mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*} \right) \geq 2^{R_0}, \\
& \left(1 + \frac{\mathbf{z}_j \mathbf{Q}_0 \mathbf{z}_j^*}{N_0 + \mathbf{z}_j \mathbf{Q}_2 \mathbf{z}_j^* + \mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*} \right) \geq 2^{R_0}, \\
& \mathbf{Q}_0 \succeq \mathbf{0}, \quad \mathbf{Q}_1 \succeq \mathbf{0}, \quad \mathbf{Q}_2 \succeq \mathbf{0}, \\
& \text{trace}(\mathbf{Q}_0) + \text{trace}(\mathbf{Q}_1) + \text{trace}(\mathbf{Q}_2) \leq P_T. \quad (35)
\end{aligned}$$

Further, we rewrite the innermost minimization in (34), namely,

$$\begin{aligned}
& \min_{\substack{k=1,2,\dots,K \\ j=1,2,\dots,J}} \left(\frac{N_0 + \mathbf{h}_k \mathbf{Q}_2 \mathbf{h}_k^* + \mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*}{N_0 + \mathbf{h}_k \mathbf{Q}_2 \mathbf{h}_k^*} \right) \\
& \left(\frac{N_0 + \mathbf{z}_j \mathbf{Q}_2 \mathbf{z}_j^*}{N_0 + \mathbf{z}_j \mathbf{Q}_2 \mathbf{z}_j^* + \mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*} \right), \quad (36)
\end{aligned}$$

in the following equivalent maximization form:

$$\begin{aligned}
& \max_{u, v} uv \quad (37) \\
& \text{s.t. } \forall k = 1, 2, \dots, K, \quad \forall j = 1, 2, \dots, J, \\
& u(N_0 + \mathbf{h}_k \mathbf{Q}_2 \mathbf{h}_k^*) \\
& - (N_0 + \mathbf{h}_k \mathbf{Q}_2 \mathbf{h}_k^* + \mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*) \leq 0, \\
& v(N_0 + \mathbf{z}_j \mathbf{Q}_2 \mathbf{z}_j^* + \mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*) \\
& - (N_0 + \mathbf{z}_j \mathbf{Q}_2 \mathbf{z}_j^*) \leq 0. \quad (38)
\end{aligned}$$

Substituting the above maximization form in (34), we get the following single maximization form:

$$\begin{aligned}
& \max_{\mathbf{Q}_0, \mathbf{Q}_1, \mathbf{Q}_2, u, v} uv \quad (39) \\
& \text{s.t. } \forall k = 1, 2, \dots, K, \quad \forall j = 1, 2, \dots, J, \\
& (2^{R_0} - 1)(N_0 + \mathbf{h}_k \mathbf{Q}_2 \mathbf{h}_k^* + \mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*) \\
& - (\mathbf{h}_k \mathbf{Q}_0 \mathbf{h}_k^*) \leq 0, \\
& (2^{R_0} - 1)(N_0 + \mathbf{z}_j \mathbf{Q}_2 \mathbf{z}_j^* + \mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*) \\
& - (\mathbf{z}_j \mathbf{Q}_0 \mathbf{z}_j^*) \leq 0, \\
& u(N_0 + \mathbf{h}_k \mathbf{Q}_2 \mathbf{h}_k^*) \\
& - (N_0 + \mathbf{h}_k \mathbf{Q}_2 \mathbf{h}_k^* + \mathbf{h}_k \mathbf{Q}_1 \mathbf{h}_k^*) \leq 0, \\
& v(N_0 + \mathbf{z}_j \mathbf{Q}_2 \mathbf{z}_j^* + \mathbf{z}_j \mathbf{Q}_1 \mathbf{z}_j^*) \\
& - (N_0 + \mathbf{z}_j \mathbf{Q}_2 \mathbf{z}_j^*) \leq 0, \\
& \mathbf{Q}_0 \succeq \mathbf{0}, \quad \mathbf{Q}_1 \succeq \mathbf{0}, \quad \mathbf{Q}_2 \succeq \mathbf{0}, \\
& \text{trace}(\mathbf{Q}_0) + \text{trace}(\mathbf{Q}_1) + \text{trace}(\mathbf{Q}_2) \leq P_T. \quad (40)
\end{aligned}$$

From the constraints in (40), it is obvious that the upper bound for u can be taken as $(1 + \min_{k=1,2,\dots,K} \frac{P_T \|\mathbf{h}_k\|^2}{N_0})$ and we denote it by u_{max} . Similarly, the upper bound for v can be taken as 1 and we denote it by v_{max} . We denote the optimum value of the optimization problem (39) by $u_{opt}v_{opt}$.

For positive secrecy rate, $u_{max} \geq u_{opt} > 1$, $v_{max} \geq v_{opt} > 0$ and $u_{opt}v_{opt} > 1$. We obtain $u_{opt}v_{opt}$ sequentially by increasing u from 1 towards u_{max} in discrete steps of size $\Delta u = (u_{max} - 1)/M$, where M is a large positive integer, and finding the maximum v such that the constraints in (40) are feasible and the product uv is maximum. The algorithm to obtain $u_{opt}v_{opt}$ as follows.

-
1. **for** ($i = 1 : 1 : M$)
 2. **begin**
 3. $u_i = 1 + (i * \Delta u)$
 4. $v_i = \max_{\substack{\mathbf{Q}_0, \mathbf{Q}_1, \mathbf{Q}_2, v, \\ u=u_i}} v$ s.t. to all constraints in (40).
 5. **if** ($i = 1$) **then** $u_{opt} = u_i$, $v_{opt} = v_i$
 6. **elseif** ($u_{opt}v_{opt} \leq u_i v_i$) **then** $u_{opt} = u_i$, $v_{opt} = v_i$
 7. **else** $u_{opt} = u_{opt}$, $v_{opt} = v_{opt}$
 8. **endif**
 9. **end for loop**
-

The constrained maximization problem in the for loop can be solved using the bisection method by checking the feasibility of the constraints in (40) at $u = u_i$ and v in the interval $[0, v_{max}]$. Having obtained $u_{opt}v_{opt}$, the secrecy rate is given by $R_1 = \log_2 u_{opt}v_{opt}$.

We can take the rank-1 approximation of \mathbf{Q}_1 and \mathbf{Q}_0 as discussed in subsection III-B, i.e., by substituting $\mathbf{Q}_0 = P_0 \phi^0 \phi^{0*}$ and $\mathbf{Q}_1 = P_1 \phi^1 \phi^{1*}$ in the optimization problem (39) and solving for P_0 , P_1 , \mathbf{Q}_2 , u and v .

IV. RESULTS AND DISCUSSIONS

We present the numerical results and discussions in this section. We obtained the secrecy rate results through simulations for $N = 2$, $K = 1$ and $J = 1, 2, 3$ eavesdroppers. The following complex channel gains are taken in the simulations: $\mathbf{h} = [2.0824 - 1.7215i, 0.0788 - 0.0583i]$, $\mathbf{z}_1 = [-0.3989 - 0.0923i, -0.6770 + 0.3371i]$, $\mathbf{z}_2 = [0.0910 - 0.8258i, 0.6642 - 0.3257i]$, $\mathbf{z}_3 = [-0.2784 - 1.3995i, -1.4867 + 0.9877i]$.

Figure 2(a) shows the secrecy rate plots for MISO broadcast channel as a function of total transmit power (P_T) when no artificial noise is injected. The secrecy rates are plotted for the cases of with and without W_0 . For the case with W_0 , the information rate of W_0 is fixed at $R_0 = 1$. From Fig. 2(a), we observe that, for a given number of eavesdroppers, the secrecy rate degrades when W_0 is present. Also, the secrecy rate degrades for increasing number of eavesdroppers. Figure 2(b) shows the R_1 vs R_0 tradeoff, where R_1 is plotted as a function of R_0 for $K = 1$, $J = 1, 2, 3$ at a fixed total power of $P_T = 9$ dB and no artificial noise. It can be seen that as R_0 is increased, secrecy rate decreases. This is because the available transmit power for W_1 decreases as R_0 is increased. The point 2.7 (approximately) on the R_0 axis where the secrecy rate drops to zero corresponds to R_0^{max} .

Figure 3(a) shows the secrecy rate plots for MISO broadcast channel as a function of total transmit power (P_T) when artificial noise is injected. Similarly, Fig. 3(b) shows the R_1

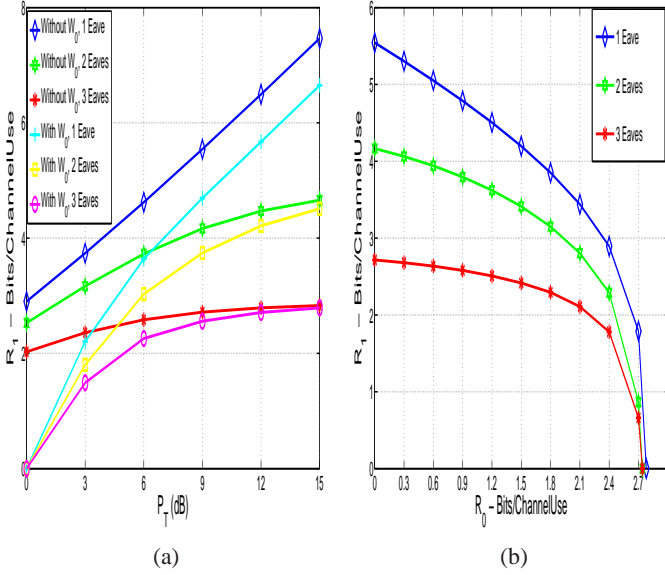


Fig. 2. Secrecy rate in MISO broadcast channel for $N = 2$, $K = 1$, $J = 1, 2, 3$, and no artificial noise. (a) secrecy rate vs total power (P_T) with/without W_0 , $R_0 = 1$; (b) R_1 vs R_0 for $P_T = 9$ dB.

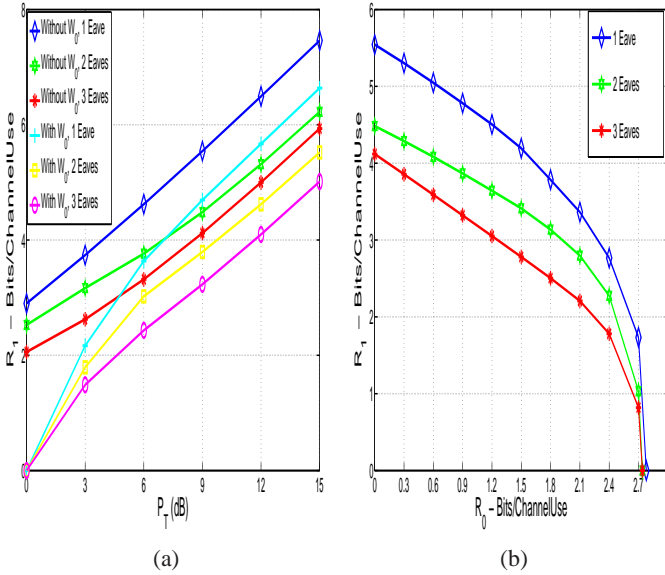


Fig. 3. Secrecy rate in MISO broadcast channel for $N = 2$, $K = 1$, $J = 1, 2, 3$, and with artificial noise. (a) secrecy rate vs total power (P_T) with/without W_0 , $R_0 = 1$; (b) R_1 vs R_0 for $P_T = 9$ dB.

vs R_0 tradeoff with artificial noise, where R_1 is plotted as a function of R_0 for $K = 1$, $J = 1, 2, 3$ at a fixed total power of $P_T = 9$ dB. We observe a significant improvement in secrecy rate as compared to Fig. 2(a) and Fig. 2(b) when $J = 2$ or 3 eavesdroppers are present. When only one eavesdropper is present, artificial noise does not help in improving the secrecy rate. This is due to the null signal beamforming by the source at the eavesdropper which is only possible when $J < N$.

Also, for the above channel conditions, we observe that the solutions Q_0 and Q_1 obtained by solving the optimization problems (19) and (39) have rank 1.

V. CONCLUSIONS

We investigated transmitter optimization problem in MISO broadcast channel with common and secret messages. The source operates under a total power constraint. It also injects artificial noise to improve the secrecy rate. We obtained the optimum covariance matrices associated with the common message, secret message, and artificial noise, which maximized the achievable secrecy rate and simultaneously met the fixed rate R_0 for the common message.

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